Assignment 4

Coverage: 15.5 in Text.

Exercises: 15.5. no 3, 4, 21, 24, 25, 27, 29, 32, 33, 38, 39. 15.6. no 9, 13, 19, 23.

Submit 15.5 no. 24, 27, 29 , 15.6 no 13, and Supplementary Problem no 3 by Oct 11.

Supplementary Problems

1. Find the equations of the planes passing through the origin and (a) $(1,2,3),(0,-2,0)$ and (b) $(0,2,-1),(3,0,5)$.

2. Find the equation of the plane passing the points $(1,0,-1),(4,0,0),(6,2,1)$.

3. Let D be a region in the plane which is symmetric with respect to the origin, that is, $(x,y)\in D$ if and only if $(-x,-y)\in D$. Show that

$$\iint_D f(x,y)\,dA(x,y)=0\ ,$$

when f is odd, that is, $f(-x,-y)=-f(x,y)$ in D. Suggestion: Use polar coordinates.

The Equation of a Plane

The equation of a plane in space is in the form

$$ax+by+cz=d\ ,$$

and $d=0$ if and only if the plane passes through the origin. Given three points in space $\mathbf{0}=(0,0,0),\mathbf{u}_1=(x_1,y_1,z_1),\mathbf{u}_2=(x_2,y_2,z_2)$, the equation of the plane can be determined by the following formula:

$$(a,b,c)=\mathbf{u}_1\times\mathbf{u}_2\ ,$$

in $ax+by+cz=0$. Here $\times$ is the cross product for vectors.

When the plane does not pass through the origin, the three points are $\mathbf{u}_0,\mathbf{u}_1,\mathbf{u}_2$. Let $\mathbf{v}_1=\mathbf{u}_1-\mathbf{u}_0,\mathbf{v}_2=\mathbf{u}_2-\mathbf{u}_0$. Then

$$(a,b,c)=\mathbf{v}_1\times\mathbf{v}_2\ ,$$

in the equation $ax+by+c=d$. The number d can be obtained by $d=ax_0+by_0+cz_0$ where $\mathbf{u}_0=(x_0,y_0,z_0)$.